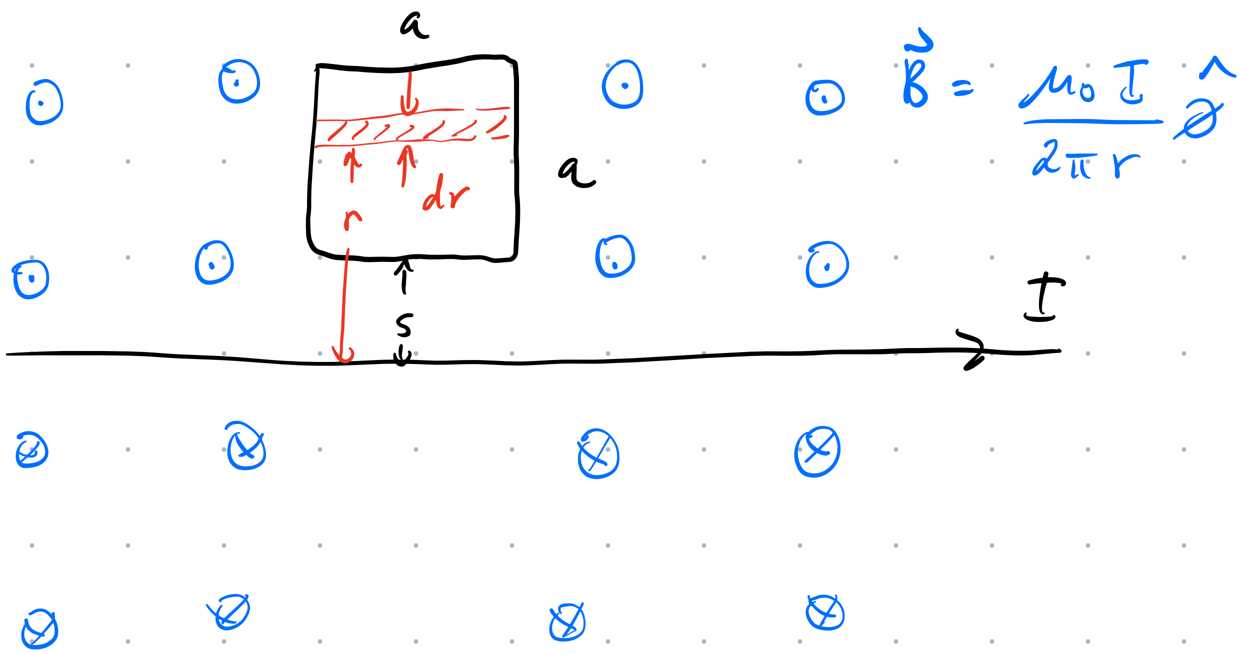
PHYS 301 Assign. #5 Sol'ns Nov. 26, 2024

1.



$$d\Phi \simeq \frac{\mu_0 I}{2\pi r} a\, dr$$

$$\therefore \Phi = \frac{\mu_0 I a}{2\pi}\int_s^{s+a}\frac{dr}{r} = \frac{\mu_0 I a}{2\pi}\ln\left(\frac{s+a}{s}\right)$$

$$\boxed{\therefore \Phi \simeq \frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{a}{s}\right)}$$

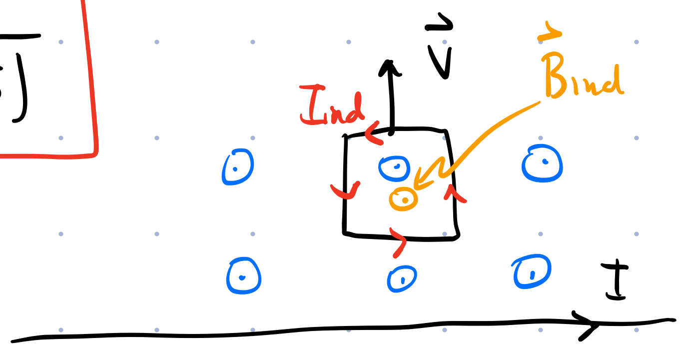
(b)

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 I a}{2\pi}\frac{d}{dt}\ln\left(1+\frac{a}{s}\right)$$

$$= -\frac{\mu_0 I a}{2\pi}\left(1+\frac{a}{s}\right)^{-1}\frac{d}{dt}\left(1+\frac{a}{s}\right)$$

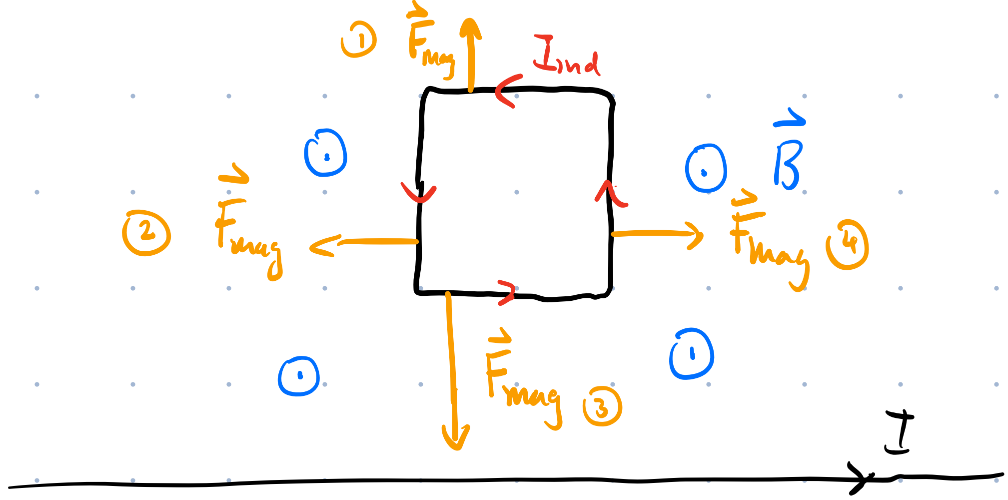
$$= \frac{\mu_0 I a}{2\pi}\left(\frac{s}{a+s}\right)\frac{a}{s^2}\underbrace{\frac{ds}{dt}}_{v}$$

$$\boxed{\mathcal{E} = \frac{\mu_0 I a^2 v}{2\pi s(a+s)}}$$



as the square loop is pulled away, the flux through the loop (directed out of the screen) decreases. To oppose this decrease in Φ, I_{ind} establishes $\vec{B}_{ind}$ which is also out of the screen. ∴, by the RHR, I_{ind} is $\boxed{\text{CCW}}$

(c)



$\vec{F}_{mag} = \vec{I}_{ind} \times \vec{B}\, a$ for each of the four sides of the loop.

The forces on sides ② & ④ vary along the lengths since $\vec{B}$ is not const. However, it doesn't matter b/c these cancel. Only need to consider sides ① & ③.

$$\therefore \vec{F}_{net} = \underbrace{\frac{\mu_0 I a^2 v}{2\pi s(a+s)R}}_{I_{ind}} \left[\underbrace{\frac{\mu_0 I}{2\pi(a+s)}}_{F_{mag}\ @\ r=a+s} - \underbrace{\frac{\mu_0 I}{2\pi s}}_{F_{mag}\ @\ r=s} \right] a\, \hat{s}$$

$$= \left(\frac{\mu_0 I a}{2\pi}\right)^2 \frac{v}{s(a+s)R} \left[\frac{1}{a+s} - \frac{1}{s} \right] a\, \hat{s}$$

$$\frac{1}{a+s} - \frac{1}{s} = \frac{s-(a+s)}{s(a+s)} = \frac{-a}{s(a+s)}$$

$$\therefore \vec{F}_{net} = -\left[\frac{\mu_0 I a}{2\pi\, s(a+s)}\right]^2 \frac{V a^2}{R}\hat{s}$$

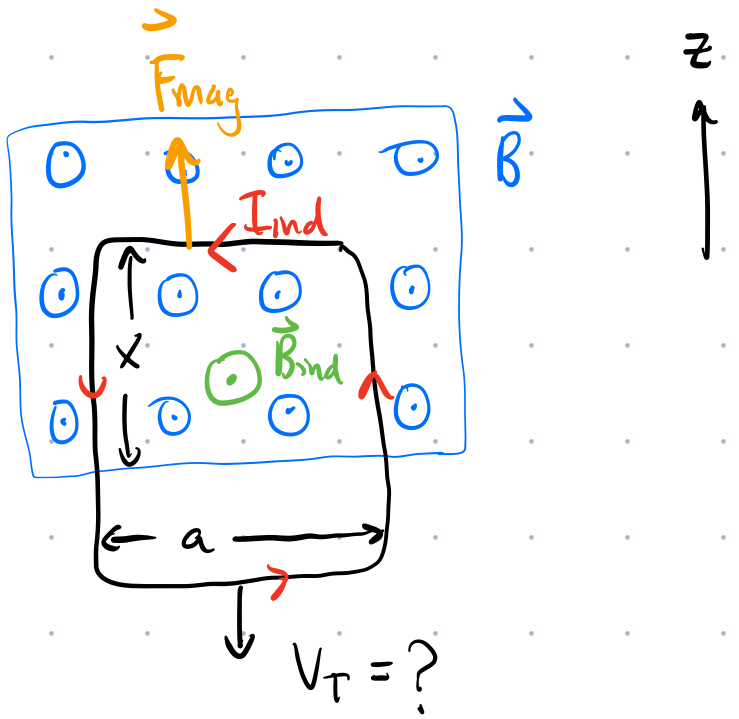
$$\boxed{\therefore \vec{F}_{net} = -\left[\frac{\mu_0 I a^2}{2\pi\, s(a+s)}\right]^2 \frac{V}{R}\hat{s}}$$

check units:

$$[F_{net}] = \left[\frac{\frac{kg\,m}{s^2 \cancel{A^2}} \cancel{A}\, \cancel{m^2}}{\cancel{m^2}}\right]^2 \frac{m}{s} \frac{s^3 A^2}{kg\, m^2}$$

$$= \frac{kg^{\cancel{2}}\, \cancel{m^2}}{s^{\cancel{4}}\, \cancel{A^2}} \frac{m}{s} \frac{\cancel{s^3}\, \cancel{A^2}}{\cancel{kg}\, \cancel{m^2}} = \frac{kg\, m}{s^2} = N \checkmark$$

2.



(a) Φ through the loop is decreasing.

At the instant shown, $\Phi = Bax$

$$\therefore \mathcal{E} = -\frac{d\Phi}{dt} = -Ba\frac{dx}{dt} = +aBV \quad \left(\text{note that } \frac{dx}{dt} < 0 \text{ since } x \text{ is decreasing}\right)$$

The induced current is ccw so that it creates $\vec{B}_{ind}$ in same dir'n as $\vec{B}$ (I chose out of the screen).

$\vec{F}_{mag}$ on the top wire of the loop points up.

$$\vec{F}_{mag} = \vec{I}_{ind} \times \vec{B} a = \frac{\mathcal{E}}{R} Ba\,\hat{z} = \frac{aBV}{R} Ba\,\hat{z}$$

$$\therefore \vec{F}_{mag} = \frac{(aB)^2 v}{R}\,\hat{z}$$

When loop reaches terminal velocity, $F_{mag} = F_g$

$$\Rightarrow \frac{(aB)^2 v_T}{R} = mg$$

$$\therefore \boxed{v_T = \frac{mgR}{(aB)^2}}$$

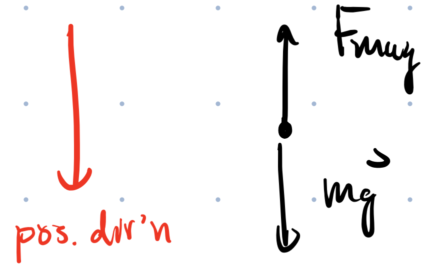
check units:

$$[v_T] = \frac{\cancel{kg}\,\frac{m}{s^2}\,\frac{\cancel{kg}\,\cancel{m^2}}{s^3\,\cancel{A^2}}}{\cancel{m^2}\left(\frac{\cancel{kg}}{s^2\,\cancel{A}}\right)^2}$$

$$= \frac{m}{s^5}s^4 = \frac{m}{s} \checkmark$$

(b) Free body diagram:

For this part of the problem, I choose the pos. dir'n to be down.



Eq'n of motion: $ma = -F_{mag} + mg$

$$\therefore m\frac{dv}{dt} = -\frac{(aB)^2}{R}v + mg$$

$$\therefore \frac{dv}{dt} = -\underbrace{\frac{(aB)^2}{mR}}_{\substack{\text{must have} \\ \text{units of } s^{-1}}} v + g$$

Define $\tau \equiv \frac{mR}{(aB)^2}$

$$\therefore \frac{dv}{dt} = -\frac{v}{\tau} + g$$

write $V = V_T + V_h$ ← homogeneous sol'n

↑ particular sol'n when $\frac{dv}{dt} = 0$

$$\therefore \frac{d}{dt}\Big(\underbrace{V_T}_{\text{const.}} + V_h\Big) = \frac{-V_T - V_h}{\tau} + g$$

$$\therefore \frac{dV_h}{dt} = -\frac{V_T}{\tau} - \frac{V_h}{\tau} + g \qquad \text{but } \frac{V_T}{\tau} = g.$$

$$\therefore \frac{dV_h}{dt} = \frac{-V_h}{\tau}$$

$$\therefore \frac{dV_h}{V_h} = \frac{-dt}{\tau} \Rightarrow \ln V_h = -\frac{t}{\tau} + A$$

$$\text{or} \quad V_h = Be^{-t/\tau} \quad \text{where } B = e^A$$

$$\therefore V = V_T + V_h = V_T + Be^{-t/\tau}$$

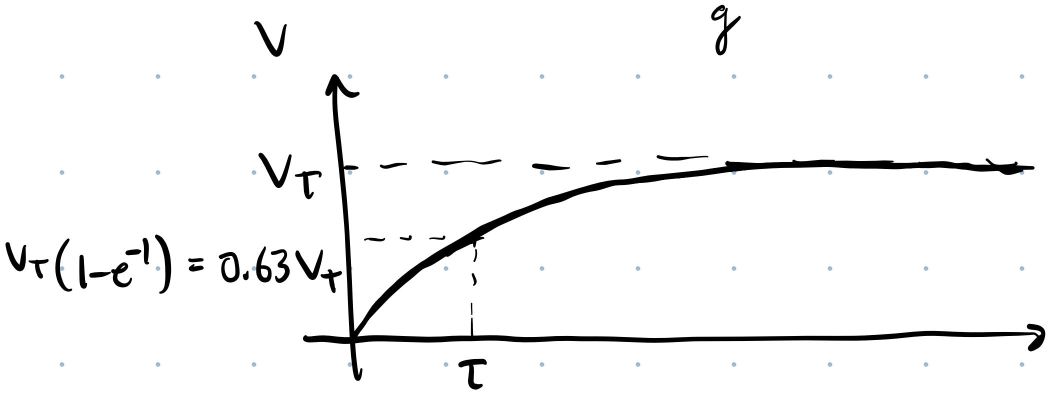
Use the initial condition $V(t=0) = 0$ to set the value of B.

$$0 = V_T + B \quad \therefore B = -V_T$$

Finally

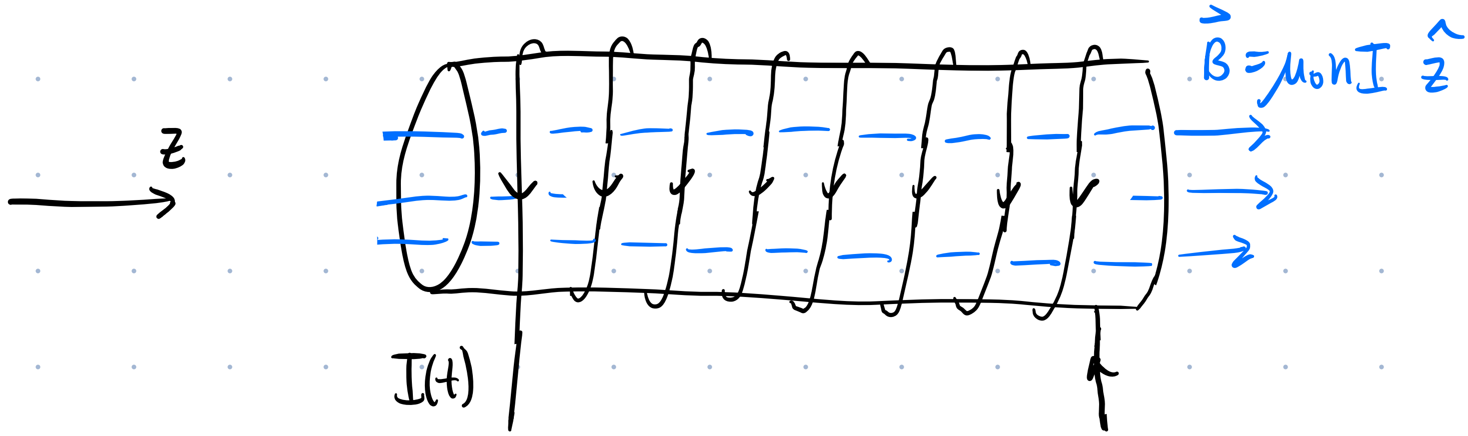
$$\boxed{V(t) = V_T\left(1 - e^{-t/\tau}\right) \quad \text{where} \quad \tau = \frac{mR}{(aB)^2}}$$

Note that $\tau = \dfrac{V_T}{g}$ $\therefore [\tau] = \dfrac{m}{s}\dfrac{s^2}{m} = s$ ✓



Same curve as a charging capacitor.

3.



Faraday's Law:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

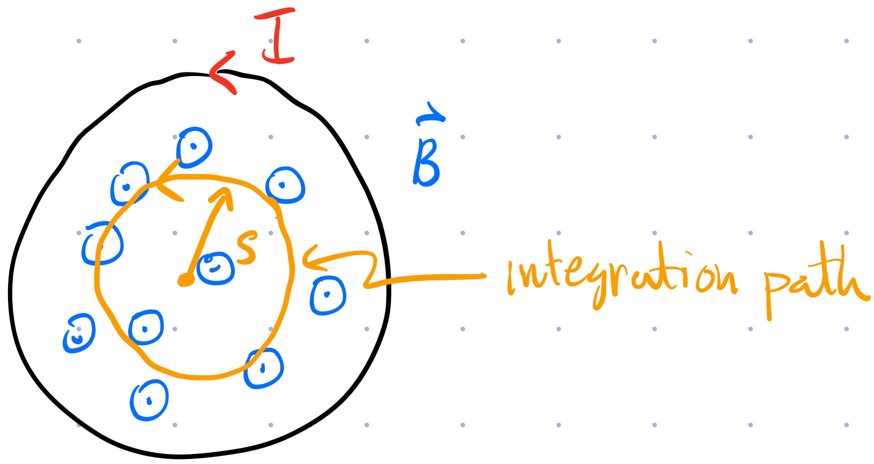
Convert to integral form:

$$\underbrace{\int \vec{\nabla} \times \vec{E} \cdot d\vec{a}}_{\text{Apply Stoke's Th.}} = -\frac{\partial}{\partial t} \underbrace{\int \vec{B} \cdot d\vec{a}}_{\Phi}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$

(a) $s < a$

End view of solenoid



Enclosed flux is $B\pi s^2 = \mu_0 n I \pi s^2$

$$\therefore -\frac{d\Phi}{dt} = -\mu_0 n \pi s^2 \frac{dI}{dt}$$

$$\oint \vec{E} \cdot d\vec{l} = E\, 2\pi s$$

$$\therefore E\, 2\cancel{\pi}\cancel{s} = -\mu_0 n \cancel{\pi} s^{\cancel{2}} \frac{dI}{dt}$$

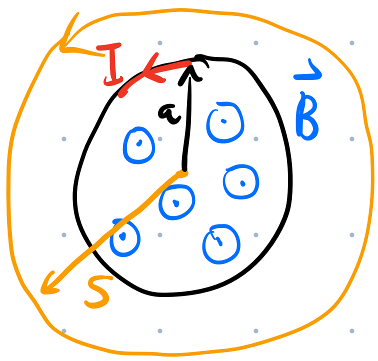
$$\therefore E = -\frac{\mu_0 n s}{2} \frac{dI}{dt}$$

or $\boxed{\vec{E} = -\frac{\mu_0 n s}{2}\frac{dI}{dt}\hat{\phi} \qquad s < a}$

If $\frac{dI}{dt}$ is pos. (I increasing),

$\vec{E}$ is in opp. dir'n of I.

(b) $s > a$



$$\oint \vec{E}\cdot d\vec{l} = E\,2\pi s \text{ as in (a)}$$

However, now $\Phi = \mu_0 n I \pi a^2$

& $\frac{d\Phi}{dt} = \mu_0 n \pi a^2 \frac{dI}{dt}$

$$\therefore E\,2\cancel{\pi} s = -\mu_0 n \cancel{\pi} a^2 \frac{dI}{dt}$$

$$\boxed{\therefore\ \vec{E} = -\frac{\mu_0 n a^2}{2s}\frac{dI}{dt}\hat{\phi}}$$

4. (a)

Faraday's Law:

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

If $\vec{E}=0$ inside a perfect conductor,

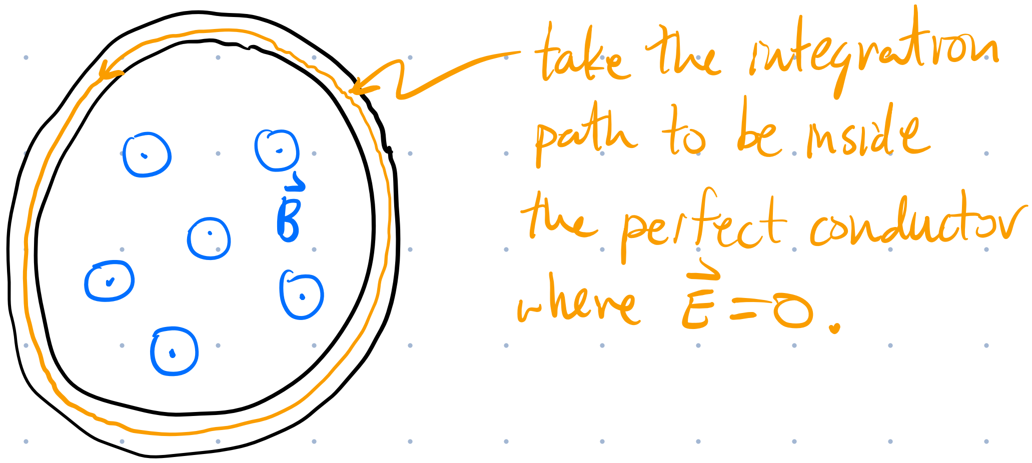
then $\vec{\nabla}\times\vec{E}=0$ s.t. $\frac{\partial\vec{B}}{\partial t}=0$

or $\boxed{\vec{B}=\text{const.}}$

(b) The integral form of Faraday's Law is:

$$\oint\vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$$

Top view of loop made of a perfect conductor



$$\oint \underbrace{\vec{E}}_{0} \cdot d\vec{l} = 0 \quad \therefore \quad 0 = -\frac{d\Phi}{dt}$$

$$\text{or} \quad \boxed{\Phi = \text{const}}$$

(c) The "fixed" version of Ampère's Law is:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \underbrace{\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\text{displacement current density.}}$$

If inside a superconductor, $\vec{B}=0$ & $\vec{E}=0$, then we're left with:

$$0 = \mu_0 \vec{J} + 0$$

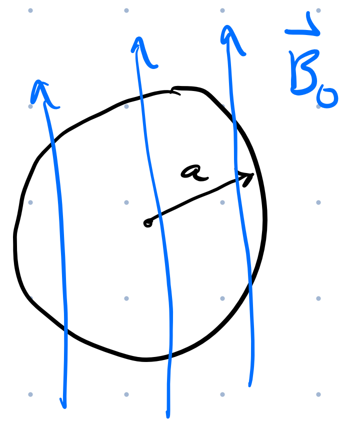
$$\therefore \vec{J} = 0$$

No current density within the bulk material of a superconductor. $\therefore$ All current must reside on the surface.

(d)

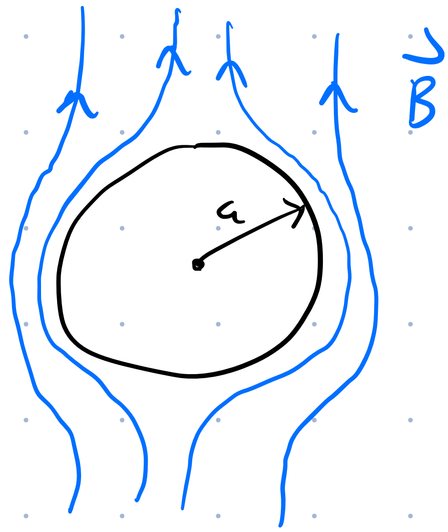
$$T > T_c$$

In this case, external fields pass through the material.



$T < T_c$

Magnetic fields expelled from superconductor



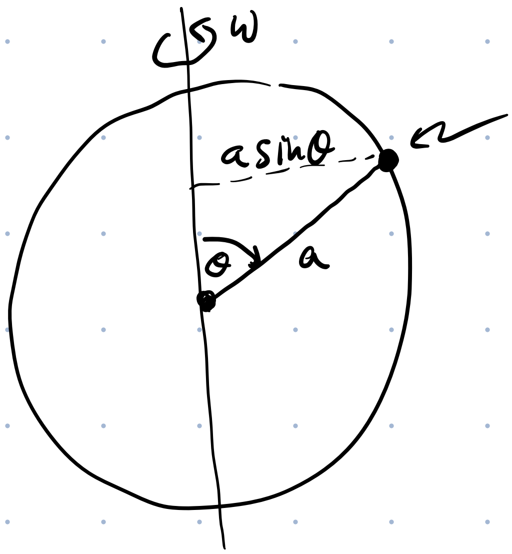
Know that we will induce currents on surface of the superconductor. These currents must create a magnetic field inside the sphere that exactly cancels $B_0 \hat{z}$.

$\therefore$ Require a <u>constant</u> magnetic field inside the sphere. Know from Assign. #4 1(f) & Tutorial # 8 3(a), that a spinning charged spherical shell creates a uniform magnetic field inside.

Specifically,

$$\vec{B} = \frac{2}{3}\mu_0 a \omega \sigma \hat{z} \quad (\#)$$

But this is equiv. to a stationary sphere with the appropriate surface current $\vec{K}$.



this pt. moves w/ speed $V = \omega a \sin\theta$ into the screen.

$$\vec{K} = \sigma \vec{V} = \sigma \omega a \sin\theta \, \hat{\phi}$$

From (#) $\quad \sigma \omega a = \dfrac{3B_0}{2\mu_0}$

$$\therefore \vec{K} = \frac{3B_0}{2\mu_0} \sin\theta \, \hat{\phi}$$

This would produce $\vec{B} = B_0 \hat{z}$ inside sphere. We want the opposite.

$$\boxed{\therefore \vec{K} = -\frac{3B_0}{2\mu_0} \sin\theta \, \hat{\phi}}$$